Learning Explainable Representations of Complex Game-playing Strategies

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Abstract

As part of learning to play complex games, human players develop a web of interconnected concepts known as a mental model. This mental model is applied to explain the game's behavior, and to inform a player's own actions in-game. Players refine their mental model of the game through deliberate practice and instruction, but this requires time and effort. In this paper, we present the problem of strategy synthesis as a means to automatically learn explainable representations of a game's strategies, that can align with a player's mental model to improve it. We present two approaches to strategy synthesis: one for learning rule-based chess strategies, and another for learning programmatic strategies via program synthesis. We evaluate our approaches by measuring the quality of learned strategies in terms of their in-game performance, and show that both methods learn strategies that produce competitive gameplay.

1. Introduction

Researchers in varied fields agree that people create *mental models* as shorthand for understanding the human experience (Boyan & Sherry, 2011). Mental models are described as "mechanisms whereby humans are able to generate descriptions of system purpose and form, as explanations of system functioning and observed system states, and as predictions (or expectations) of future system states" (Rouse et al., 1992). In the context of games, players refine their mental models through solitary practice or formal instruction, discussion of strategies among peers, or investigation of underlying game mechanics (Boyan et al., 2018). Evidence suggests that when provided with optimal game strategies, players are able to transfer them to their game play (Paredes-Olay et al., 2002). However, acquiring good strategies involves many hours of deliberate practice, or access to expert knowledge, which may not be easily available. Automatically learning good game-playing strategies that players can use to refine their mental model would help overcome this challenge.

The problem of *strategy synthesis* can be described as the automatic learning of a game-playing strategy, in some suitably interpretable representation, that can be understood by a player to change their mental model and measurably improve their performance in-game. Existing work focuses mostly on the computational learning algorithms and representations of strategies. For example, Butler et al. (2017) are able to learn strategies using constraint satisfaction as condition-action rules to solve the puzzle game of Nonograms. Canaan et al. (2018) use a genetic algorithm to learn strategies represented using domain-specific rules for the card game *Hanabi*. A recent line of work models a strategy as an executable program in a domain-specific programming language (DSL), which can then be learned by applying techniques from program synthesis (de Freitas et al., 2018; Mariño et al., 2021; Mariño & Toledo, 2022; Medeiros et al., 2022). Considerably less attention has been devoted to investigating how well the learned strategies can be understood by players, and which factors contribute to their interpretability.

Our contributions in this work are two-fold. First, we present a cognitively-inspired strategy model for chess that is based on existing models of chess strategy found in the game's literature. We present a learning algorithm and metrics to evaluate the effectiveness of a strategy. We show that the learned strategies can approximate a human beginner better than a random baseline. Second, we present a learning algorithm for programmatic representations of strategies based on the transformer model (Vaswani et al., 2017). We show that it is competitive with prior state-of-the-art in terms of maximizing reward while being much more sample efficient.

2. Strategy Synthesis

We define strategy synthesis through the lens of reinforcement learning (RL). RL seeks to find an optimal policy $\pi_*(\cdot, \theta)$ parametrized by a model θ that maximizes the expected return in a Markov Decision Process (MDP) $\langle \mathcal{S}, \mathcal{A}(s), \mathcal{P}, \mathcal{R}, \gamma \rangle$. Our definitions of these terms are based on the treatment by Sutton & Barto (2018).

Strategy synthesis attempts to solve the same problem as RL, with the strategy model being the parametrization θ of a policy, and the synthesis algorithm being the optimization procedure. Unlike RL, strategy synthesis has the additional constraint that the learned strategy model must be *aligned* with the mental model of the player. Model matching theory posits that the "alignment of game models and external situations can facilitate the player's transfer of mental models between games and external situations" (Boyan et al., 2018). Alignment is framed as a matter of creating accurate facsimiles of the game, and providing the necessary tools and scaffolds to help players "make sense of" the strategy (Martinez-Garza & Clark, 2017). However, it is unclear how alignment may be meaningfully evaluated.

As a substitute for alignment between a learned strategy and a player's mental model, we propose using computational metrics for model *interpretability* derived from the field of explainable AI.

Interpretability is defined as "the ability to not only extract or generate explanations for the decisions of the model, but also to present this information in a way that is understandable by human (non-expert) users to, ultimately, enable them to predict a model's behaviour" (Puiutta & Veith, 2020). Some metrics that have been found to correlate with interpretability are the number of cognitive chunks used and the model size (Lage et al., 2019). However, we do not measure the interpretability of the learned strategies in this work.

3. Learning Chess Strategies

In this section, we describe our strategy model for chess inspired by the concept of *chess tactics*. We present a learning algorithm based on inductive logic programming that learns chess strategies from gameplay data. We evaluate the learned strategies and show that our algorithm can learn strategies that approximate a human beginner better than a random baseline.

3.1 Chess Strategy Model

We model a chess strategy as a first-order logic rule expressed in Prolog (Wielemaker, 2003) using a domain-specific predicate vocabulary $\mathcal{P}$. As seen in Figure 1, the rule head of the strategy consists of the variables `Position`, `From` and `To`. The input state is described by `Position`, which is also expressed in first-order logic using $\mathcal{P}$. `From` and `To` describe the output action, namely, the move which begins from the square `From` and ends on the square `To`.

```
tactic(Position, From, To) ←
    feature_1 (···),
    feature_2 (···),
    ⋮
    feature_n (···)
```

Figure 1: Our chess strategy model expressed in Prolog pseudocode. Every `feature_i` clause is a rule defined in the predicate vocabulary.

Our usage of first-order logic to model chess tactics is motivated by the following reasons —

1. Chess tactics are an important concept that human players use to think about chess (Szabo, 1984) and are useful in chess education (Gobet & Jansen, 2006).
2. First-order logic has been extensively used to model chess patterns (Berliner, 1975; Pitrat, 1977; Wilkins, 1979; Huberman, 1968; Bramer, 1977; Bratko, 1982).
3. Logic rules are commonly acknowledged to be interpretable and have a long history of research (Zhang et al., 2021).

To evaluate the learned strategies, we define the metrics of *coverage* and *divergence* for a strategy σ on a set of positions P as follows —

$$\text{Coverage}(\sigma, P) \doteq \frac{|P_\text{A}|}{|P|} \tag{1}$$

$$\text{Divergence}_E(\sigma(\cdot), P) \doteq \frac{1}{|P_\text{A}|} \sum_{(s,a_1)\in P_\text{A}} \sum_{a_2 \in \mathcal{A}(s)} \sigma(a_2|s) d_E(s, a_1, a_2) \tag{2}$$

where P_A is the set of positions in P where the strategy σ is applicable, and d_E is a distance measure between actions a_1 and a_2 in state s.

3.2 Learning Chess Strategies using ILP

Inductive logic programming (ILP) is a form of symbolic machine learning that aims to induce a hypothesis (a set of logical rules) that generalizes given training examples. (Cropper & Dumančić, 2022). An ILP problem is specified by three sets of Horn clauses —

- B, the background knowledge,
- E^+, the set of positive examples of the concept, and
- E^-, the set of negative examples of the concept.

The ILP problem is to induce a hypothesis $H \in \mathcal{H}$ (an appropriately chosen hypothesis space) that, in combination with the background knowledge, entails all the positive examples and none of the negative examples. Formally, this can be written as —

$$\forall e \in E^+, H \cup B \models e \text{ (i.e., } H \text{ is } \textit{complete)}$$
$$\forall e \in E^-, H \cup B \not\models e \text{ (i.e., } H \text{ is } \textit{consistent)}$$

We formalize the problem of learning a chess strategy as an ILP problem $\langle E^+, E^-, B\rangle$. E^+ is a set of $\langle\text{position}, \text{move}\rangle$ tuples where the move made in the position is drawn from a target policy. E^- is a set of $\langle\text{position}, \text{move}\rangle$ tuples where the move made in the position is *not* drawn from the target policy. B is the predicate vocabulary used to express positions, moves and chess strategies. We learn a hypothesis H that maximizes the number of examples entailed in E^+ and minimizes the number of examples entailed in E^-. The hypothesis H is our chess strategy σ.

To solve this ILP problem, we use a Prolog-based ILP system called Popper (Cropper & Morel, 2021). We modify Popper to constrain it to produce legal moves and to prune strategies that are never applicable or whose recall on the training set is less than an empirically-determined threshold.

3.3 Evaluation

To evaluate our system for chess strategy synthesis, we trained it using a dataset of 1297 (position, move) pairs sampled from online games played by beginner players, and evaluated it on a held-out

```
fork(Position,From,To) ←
    make_move(From, To, Position, NewPosition),
    attacks(To, Square1, NewPosition),
    attacks(To, Square2, NewPosition),
    different_pos(Square1, Square2),
```

Figure 2: An interpretation of the *fork* tactic from the chess literature using our predicate vocabulary. The first `attacks` clause states that the piece at `To` attacks the opposing piece at `Square1` in the current position.

set of 100 pairs using the metrics of accuracy, coverage and divergence. To serve as the oracles for calculating the distance metric, we used the chess engines Stockfish 14 (Romstad et al., 2021) and Maia-1600 (McIlroy-Young et al., 2020). As baseline strategies, we used the aforementioned chess engines, and the random strategy. We report the results for each of accuracy, coverage and divergence as a histogram with 20 buckets. We see that when using Stockfish 14 as a distance metric, the learned strategies are better at imitating the training set than a random baseline.

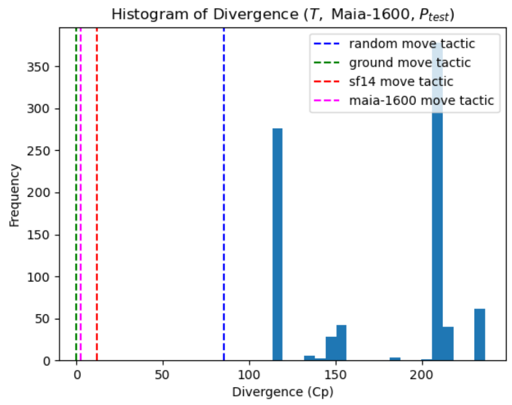
4. Learning Programmatic Strategies with Transformers

In this section, we present a novel method for learning programmatic strategies using transformers that is competitive with state-of-the-art methods while being more sample efficient. We first describe how problem of synthesizing programmatic strategies can be modeled as a reinforcement learning problem. We then present our method, which applies the decision transformer model to discrete environments. We evaluate our method using tasks in a grid-based programming environment and show that the strategies learned by our method are competitive with a state-of-the-art programmatic policy synthesis algorithm, while being much more sample efficient.

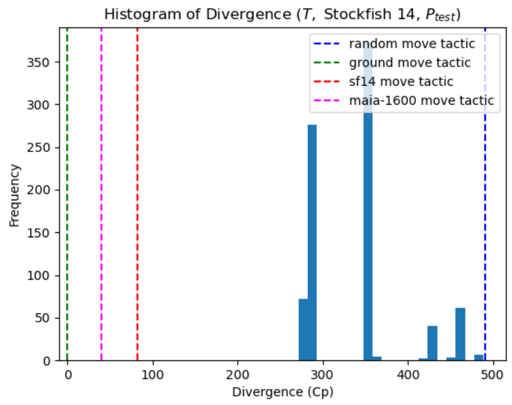
4.1 Programmatic Policy Synthesis as a Reinforcement Learning Problem

Programmatic policy synthesis is the problem of approximating a target policy using a program, as specified by some DSL. We cast this as a RL problem by modeling the program synthesis environment as an MDP. Formally, given a DSL $G = (V, \Sigma, R, S)$ (Sipser, 1996) which defines the space of possible programmatic policies, and an MDP $\mathcal{M}_{\text{exec}}$ in which a programmatic policy σ can execute and obtain reward, we define the programmatic policy synthesis task as that of finding an optimal policy in the MDP $\mathcal{M}_{\text{syn}}$, where:

- $\mathcal{S}_{\text{syn}} \doteq \{w \in (V \cup \Sigma)^* \mid S \overset{*}{\Rightarrow} w\}$ and is the current sequence of terminals and non-terminals in the partially expanded program $(w_0, w_1, \cdots, w_i, \cdots)$.
- $\mathcal{A}_{\text{syn}} \doteq R \times V \times \mathbb{N}$ and is the space of all actions consisting of applying a rule r to a non-terminal symbol v located at index i in the current state.

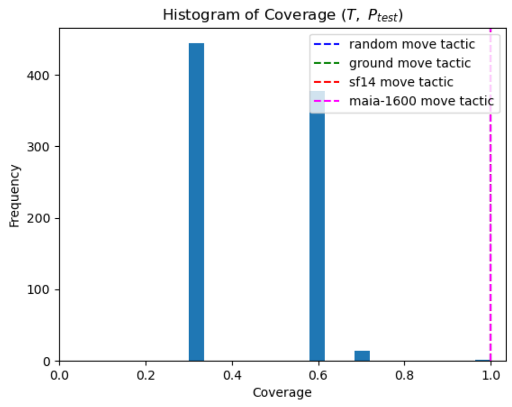


(a) Divergence histogram for T evaluated using Maia-1600

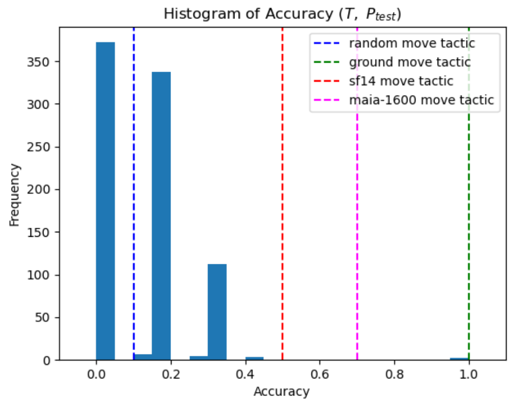


(b) Divergence histogram for T evaluated using Stockfish 14

Figure 3: Divergence histograms for T



(a) Coverage histogram for T



(b) Accuracy histogram for T

- $\mathcal{P}_{\text{syn}}$ is the transition function which represents the deterministic transition to a state where the production rule has been applied to the selected non-terminal symbol if such an action was valid. If not, the state remains unchanged.
- $d_{0,\text{syn}}(S) = 1$
- $r_{\text{syn}} \doteq \begin{cases} r_{\text{exec}}, & \text{if current state } s \in \mathcal{S}_{\text{syn}} \text{ is terminal} \\ 0, & \text{otherwise} \end{cases}$
- $\gamma_{\text{syn}} = 1$

Given this formulation, we can learn strategy models in the form of a program (in some DSL) using an RL algorithm.

4.2 Applying Decision Transformers to Programmatic Policy Synthesis

Decision transformers were proposed by Chen et al. (2021) as an architecture to apply the sequence-modeling capabilities of transformers (Vaswani et al., 2017) to sequential decision problems. Trajectories τ are represented as:

$$\tau = \left(\widehat{R}_1, s_1, a_1, \widehat{R}_2, s_2, a_2, \cdots, \widehat{R}_T, s_T, a_T\right) \tag{3}$$

where rewards r_t are modeled as the returns-to-go $\widehat{R}_t = \sum_{t'=t}^{T} r_{t'}$. States, actions and returns are embedded using separate, fully-connected layers. Additionally, an embedding is learned for each timestep and added to each token. This is different from the standard positional embedding used by transformers, as one timestep corresponds to three tokens. The tokens are then processed by a GPT (Radford et al., 2018) model, which predicts future actions via autoregressive modeling.

The original decision transformer architecture cannot handle environments with discrete actions like $\mathcal{M}_{\text{syn}}$, and hence we make the following modifications to the architecture:

- **Action Masking**: To ensure that the model does not sample invalid actions, we mask out invalid actions by following the approach described in Huang & Ontañón (2022). Informally, given the unnormalized logits representing action predictions, we mask out invalid actions by setting their logits to $-\infty$ and apply a softmax to obtain a probability distribution.
- **Cross-entropy loss**: The original architecture used a simple L2 loss since actions were continuous. For discrete actions, we use a cross-entropy loss as is usual with classification tasks (Bishop, 2006).
- **Sampling from action distribution**: The original architecture uses a regression layer to predict the continuous-valued action. Since we are dealing with discrete actions, we sample from the predicted action distribution to obtain the action.

Additionally, to better represent the sequential nature of states in the MDP, we embed the state using a sequential model, specifically a GRU (Cho et al., 2014). This follows from existing approaches to sentence embedding for information retrieval (Palangi et al., 2016; Kiros et al., 2015; Le & Mikolov, 2014). We empirically find that this improves the model's performance when compared to embedding the state directly [1].

4.3 Karel Domain

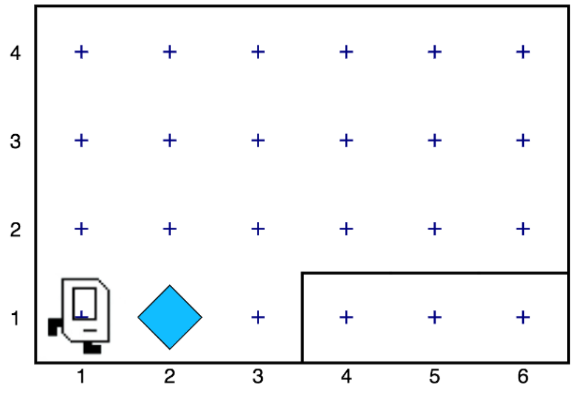


Figure 5: A sample Karel world of size 4×6. The blue diamond represents a marker. The agent cannot travel through walls.

Karel is a simple programming language designed for teaching programming to beginners (Pattis, 1994). Programs written in Karel are used to control an agent that can move around a grid world and interact with markers in the world. The agent has actions for moving and interacting with markers, and perceptions for detecting obstacles and markers. Trivedi et al. (2021) introduce six different tasks for the Karel domain with different goals and reward structures. The initial configuration for these tasks are randomly sampled for each episode reset. They also introduce a DSL to represent programmatic strategies for solving these tasks (see Figure 6).

4.4 Evaluation

To investigate if the decision transformer can perform well on the programmatic policy synthesis task, we evaluate it using the Karel environment and tasks. We train the decision transformer using a dataset of 50,000 programs contributed by Trivedi et al. (2021). We use the same hyperparameters and training procedure as in Chen et al. (2021). A learned strategy's reward is measured as its average return over 10 random initial states. We compare the performance of strategies learned using the decision transformer with those learned by using LEAPS Trivedi et al. (2021). We also compare the number of candidate programs searched by each method. From the results in Table table 1, we see that the decision transformer is competitive with LEAPS in terms of performance, while being more sample efficient.

1. The code release for this paper can be found at `https://github.com/AbhijeetKrishnan/decision-transformer` .

```
⟨program⟩ ::= 'DEF' 'run' 'm(' ⟨stmt⟩ 'm)'

⟨stmt⟩ ::= 'REPEAT' ⟨cste⟩ 'r(' ⟨stmt⟩ 'r)'
  | ⟨stmt⟩ ⟨stmt⟩
  | ⟨action⟩
  | 'IF' 'c(' ⟨cond⟩ 'c)' 'i('⟨stmt⟩ 'i)'
  | 'IFELSE' 'c(' ⟨cond⟩ 'c)' 'i(' ⟨stmt⟩ 'i)' 'e(' ⟨stmt⟩ 'e)'
  | 'WHILE' 'c(' ⟨cond⟩ 'c)' 'w(' ⟨stmt⟩ 'w)'

⟨cond⟩ ::= 'not' 'c(' ⟨cond_without_not⟩ 'c)'
  | ⟨cond_without_not⟩

⟨cond_without_not⟩ ::= 'frontIsClear'
  | 'leftIsClear'
  | 'rightIsClear'
  | 'markersPresent'
  | 'noMarkersPresent'

⟨action⟩ ::= 'move'
  | 'turnLeft'
  | 'turnRight'
  | 'putMarker'
  | 'pickMarker'

⟨cste⟩ ::= 'R='⟨int⟩
```

Figure 6: The domain-specific language (DSL) for constructing programs in the Karel environment.

Table 1: Mean return ($[0, 1.1]$) of the best program found and number of unique programs explored by LEAPS and the Decision Transformer model on each Karel task. The standard deviation is reported in parentheses.

Task	**Mean Return**		**Unique Programs**	
	LEAPS	DT	LEAPS	DT
cleanHouse	0.16 (0.13)	0.23	3627	59
fourCorners	0.35 (0.00)	0.35	9872	55
harvester	0.61 (0.21)	0.66	11708	28
randomMaze	0.97 (0.04)	1.0	295	63
stairClimber	0.74 (0.49)	1.1	298	49
topOff	0.80 (0.11)	0.66	30278	63

5. Related Work

Neural program synthesis Program synthesis is the task of automatically finding a program in the underlying programming language that satisfies the user intent expressed in some form of specification (Gulwani et al., 2017). Examples of specifications include formal rules (Manna & Waldinger, 1980), input/output pairs (Neelakantan et al., 2016; Devlin et al., 2017; Gaunt et al., 2017; Shin et al., 2018; Bunel et al., 2018; Lázaro-Gredilla et al., 2019; Chen et al., 2019; Yang et al., 2021), demonstrations (Sun et al., 2018; Xu et al., 2018; Burke et al., 2019), natural language instructions/prompts (Liang et al., 2023) and MDP rewards (Li et al., 2020; Trivedi et al., 2021; Qiu & Zhu, 2022). Neural program synthesis focuses specifically on applying statistical learning methods based on neural networks to the problem of program synthesis. We attempt to synthesize programs from MDP rewards using a transformer-based approach to the task. This is motivated by the empirical success of the transformer model on sequence learning tasks.

Learning programmatic policies A reinforcement learning policy's *interpretability* refers to the ability of a human to understand, generate explanations for and predict the policy's behavior (Puiutta & Veith, 2020). A result-based approach towards learning interpretable policies is by learning a *surrogate model* that approximates the original model with a simpler, ante-hoc explainable one (Speith, 2022). A line of work focuses on learning surrogate models in the form of programs (in some programming language) (Orfanos & Lelis, 2023; Qiu & Zhu, 2022; Trivedi et al., 2021; Inala et al., 2020; Verma et al., 2018, 2019). A related line of work focuses on learning programmatic policies for game-based MDPs (Mariño et al., 2021; Mariño & Toledo, 2022; Medeiros et al., 2022). This work investigates the use of a transformer-based architecture to the same task.

6. Conclusion and Future Work

We presented the problem of strategy synthesis and explored its connections to mental model matching, model interpretability and program synthesis. We introduced two novel approaches towards strategy synthesis – a) a rule-based strategy model for chess based on chess tactics, and an associated learning method, which we showed could learn strategies that better approximated a beginner than a random baseline, and b) a transformer-based learning method for programmatic strategies that was competitive with a state-of-the-art programmatic policy learning method while being more sample-efficient.

In future work, we plan to benchmark the decision transformer, and other programmatic policy learning methods, on a wider range of game-based environments. We also plan to investigate the interpretability of the learned strategies, and to measure the extent to which they are able to measurably improve a player's performance.

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